Presented here is a more thorough explanation of the behavior of breadth-first search (bfs) than provided in the text.

Let $G$ be a graph (either directed or undirected) and let $s$ be the root a bfs-tree $T$ resulting from a bfs traversal of $G$; among all nodes in $T$, $s$ is the first discovered. We consider the traversal algorithm to be augmented to assign depth values: $\text{depth}(s) = 0$ and $\text{depth}(w) = \text{depth}(v) + 1$ where $v$ is the parent of $w$ in $T$. Define the \textit{edge-length} of a path to be the number of edges belonging to the path, and define $\text{dist}(s, v)$, the distance from $s$ to a node $v$, to be the smallest edge-length of any path from $s$ to $v$ in $G$. Our main goal is to establish the following claim.

\textbf{Claim.} There exists a path from $s$ to the node $u$ if and only if $\text{depth}(u)$ has been assigned, in which case $\text{depth}(u) = \text{dist}(s, u)$.

We use the following \textit{Observations} about breadth-first search. ($a$, $b$, $v$, $w$, and $x$ denote nodes in $T$.)

1. For a node $w$ in $T$, $w \neq s$, the parent of $w$ is the first discovered node $x$ such that there is an edge from $x$ to $w$.

2. If $a$ is the parent of $v$ and $b$ is the parent of $w$ in $T$, and $a$ is discovered before $b$, then $v$ gets discovered before $w$.

\textbf{Lemma.} For nodes $v$ and $w$, if $v$ gets discovered before $w$ then $\text{depth}(v) \leq \text{depth}(w)$.

\textbf{Proof:} We argue by contradiction. Let $v$ and $w$ be a pair of nodes that constitute a counter-example with $w$ chosen as having the least depth value among all counter-examples. To constitute a counter-example, it must be that $v$ is discovered before $w$ and $\text{depth}(v) > \text{depth}(w) \geq 0$, so that both $v$ and $w$ have parents $a$ and $b$ respectively. The nodes $a$ and $b$ are distinct, since otherwise $v$ and $w$ would have the same depth values. The second observation (above) implies that $a$ is discovered before $b$. Furthermore, based on the minimality condition for the choice of $w$, $\text{depth}(a) \leq \text{depth}(b)$. But then $\text{depth}(v) \leq \text{depth}(w)$ since $\text{depth}(v) = \text{depth}(a) + 1$ and $\text{depth}(w) = \text{depth}(b) + 1$.

\textbf{Proof of the Claim:} If $\text{depth}(u)$ is assigned, then there is a path from $s$ to $u$, namely, through the bfs-tree. We complete the proof by showing that if a path from $s$ to $u$ exists, then $\text{depth}(u)$ is assigned the value $\text{dist}(s, u)$. To the contrary let $u$ be a node having least distance from $s$, for which either $\text{depth}(u)$ is not assigned, or $\text{depth}(u) > \text{dist}(s, u)$. Certainly $u \neq s$, so let $v$ be the node immediately preceding $u$ on a path of minimal edge-length from $s$ to $u$. Then $\text{dist}(s, v) = \text{depth}(v)$ (by the minimality condition under which $u$ is chosen), and the parent of $u$ is either $v$ or a node discovered earlier (by the first observation, above). It then follows from the lemma that $\text{depth}(u) \leq \text{depth}(v) + 1 = \text{dist}(s, u)$, completing the proof.